Journal of
Thai Interdisciplinary Research
Volume 12, Number 3, Pages 54 - 60

On fuzzy sets in non-associative near rings

Pairote Yiarayong[1]

[1]Department of Mathematics, Faculty of Science and Technology,
Pibulsongkram Rajabhat University, Phitsanuloke 65000, Thailand

Abstract

The purpose of this paper is to introduce the notion of fuzzy set, fuzzy left and fuzzy right ideals in nLA-rings, and to study fuzzy left and fuzzy right ideals in LA-rings. We show that a nonempty subset I of an nLA-ring N is an nLA-subring (left ideal, right ideal, ideal) of N if and only if f_I (tf_I) is a fuzzy nLA-subring (fuzzy left ideal, fuzzy right ideal, fuzzy ideal) of N. Finally we show that if f is a fuzzy nLA-subring (fuzzy left ideal, fuzzy right ideal, fuzzy ideal) of N, then $N_f = \{x \in N : f(x) = 0\}$ is an nLA-subring (left ideal, right ideal, ideal) of N.

Keywords: nLA-ring, ideal, level, left ideal, right ideal

Article history: Received 8 December 2016, Accepted 7 June 2017

1. Introduction

The concept of fuzzy sets was first proposed by Zadeh [1] in 1965, which has a wide range of applications in various fields such as computer engineering, artificial intelligence, control engineering, operation research, management science, robotics and many more. Abou-Zaid [2] introduced the notion of a fuzzy subnear-ring, and studied fuzzy left (right) ideals of a near-ring, and gave some properties of fuzzy prime ideals of a near-ring. Narayanan and Manikantan [3] to discuss a lot on fuzzy subgroup, subnear-rings, ideals of rings and near-rings. In [4], the notion of fuzzy ideals in near-rings was introduced by Kim and Kim. Davvaz [5] has discussed the concept of fuzzy ideals of near-rings with interval valued membership functions.

It was much later when Kamran [6] in 1987 succeeded in defining a non-associative group which they called an LA-group) and can be equally manipulated with as a subtractive group. The introduction of a left almost group (simply an LA-group) is an offshoot of an LA-semigroup. LA-group is a non-associative structure with interesting properties. Yusuf in [7] introduced the concept of a left almost ring (LA-ring). That is, a nonempty set R with two binary operations "+" and "$\cdot$" is called a left almost ring, if $(R,+)$ is an LA-group, $(R,\cdot)$ is an LA-semigroup and distributive laws of "$\cdot$" over "+" hold. Further in [7] Shah and Rehman generalized the notions of commutative rings into LA-rings. However Shah and Fazal ur Rehman in [8] generalize the notion of an LA-ring into a near left almost ring. A near left almost ring (nLA-ring) N is an LA-group under "+", an LA-semigroup under "$\cdot$" and left distributive property of "$\cdot$" over "+" holds. Furthermore, in this paper we characterize the fuzzy left (right) ideals of an nLA-ring related to level left (right) ideals.

2. Preliminaries

In this section, we refer to [6, 8], for some elementary aspects and quote few definitions, and essential examples to step up this study. For more details, we refer to the papers in the references.

Definition 2.1 [6] A groupoid S is called a left almost semigroup (simply an LA-semigroup) if it satisfies the left invertive law: $(ab)c = (cb)a,$ for all $a,b,c \in S.$

Example 2.2 Define a mapping $\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}$ by $ab = b-a,$ for all $a,b\in\mathbb{Z}$ where "$-$" is a usual subtraction of integers. Then $\mathbb{Z}$ is an LA-semigroup.

Definition 2.3 [6] An LA-semigroup $(G,\cdot)$ is called a left almost group (simply an LA-group), if there exists left identity $e\in G$ (that is $ea = a,$ for all $a\in G$), for all $a\in G$ there exists $a^{-1}\in G$ such that $a^{-1}a = e = aa^{-1}$.

Definition 2.4 [8] Let $(N,+)$ be an LA-group. Then N is said to be a near left almost ring (or simply an nLA-ring), if there exists a mapping $N\times N\to N$ (the image of (x,y) is xy) satisfying the following conditions;

(1). $x(y+z) = xy + xz;$

(2). $(xy)z = (zy)x,$ for all $x,y,z\in N.$

DOI 10.14456/jtir.2017.21

*E-mail: pairote0027@hotmail.com

Example 2.5 [8] Let $(F,+,\cdot)$ be any field. Then $(F,\oplus,\odot)$ is an nLA-ring by defining the binary operations as; for $x,y\in F, x\oplus y=y-x$ and

$$x\odot y=\begin{cases}0 & ;x=0 \text{ or } y=0\\ yx^{-1} & ;\text{otherwise.}\end{cases}$$

Let N be an nLA-ring. If S is a nonempty subset of N and S is itself an nLA-ring under the binary operation induced by N, then S is called an nLA-subring of N [8]. An LA-subring I of N is called a left ideal of N if $NI\subseteq I$ and I is called a right ideal of N if for all $m,n\in N$ and $i\in I$ such that

$$(i+m)n-mn\in I$$

and I is called an ideal of N if I is both a left and a right ideal of N [8].

A function f from N to the unit interval $[0,1]$ is a fuzzy subset of N [1]. The nLA-ring N itself is a fuzzy subset of N such that $N(x)=1$ for all $x\in N$, denoted also by N. Let f and g be two fuzzy subsets of N. Then the inclusion relation $f\subseteq g$ is defined $f(x)\le g(x)$, for all $x\in N$. $f\cap g$ and $f\cap g$ are fuzzy subsets of N defined by

$$(f\cap g)(x)=min\{f(x),g(x)\},$$
$$(f\cap g)(x)=max\{f(x),g(x)\}$$

for all $x\in N$. More generally, if $\{f_\alpha:\alpha\in\beta\}$ is a family of fuzzy subsets of N, then $\bigcap_{\alpha\in\beta} f_\alpha$ and $\bigcup_{\alpha\in\beta} f_\alpha$ are defined as follows:

$$\left(\bigcap_{\alpha\in\beta} f_\alpha\right)(x)=\bigcap_{\alpha\in\beta} f_\alpha(x)=inf\{f_\alpha(x):\alpha\in\beta\},$$

$$\left(\bigcup_{\alpha\in\beta} f_\alpha\right)(x)=\bigcup_{\alpha\in\beta} f_\alpha(x)=sup\{f_\alpha(x):\alpha\in\beta\}$$

and will be the intersection and union of the family $\{f_\alpha:\alpha\in\beta\}$ of fuzzy subset of N. The product $f\circ g$ is defined as follows:

$$(f\circ g)(x)=\begin{cases}\bigcup_{x=yz} min\{f(y),g(z)\} & ;\exists y,z\in N, \text{such that } x=yz\\ 0 & ;\text{otherwise.}\end{cases}$$

A fuzzy subset f of N is called a fuzzy nLA-subring of N if $f(x-y)\ge min\{f(x),f(y)\}$ and

$$f(xy)\ge min\{f(x),f(y)\},$$

for all $x,y\in N$. A fuzzy nLA-subring f of an nLA-ring N is called a fuzzy left ideal of N if $f(xy)\ge f(y)$ for all $x,y\in N$. A fuzzy right ideal of N is a fuzzy nLA-subring f of N such that $f((x+y)z-yz)\ge f(x)$, for all $x,y,z\in N$. A fuzzy ideal of N is a fuzzy nLA-subring f of N such that $f(xy)\ge f(y)$ and $f((x+y)z-yz)\ge f(x)$, for all $x,y,z\in N$.

Example 2.6 Let $N=\{0,1,2,3,4,5,6,7\}$ be a set with two binary operations as follows:

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	2	0	3	1	6	4	7	5
2	1	3	0	2	5	7	4	6
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	6	4	7	5	2	0	3	1
6	5	7	4	6	1	3	0	2
7	7	6	5	4	3	2	1	0

·	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	4	4	0	0	4	4	0
2	0	4	4	0	0	4	4	0
3	0	0	0	0	0	0	0	0
4	0	3	3	0	0	3	3	0
5	0	7	7	0	0	7	7	0
6	0	7	7	0	0	7	7	0
7	0	3	3	0	0	3	3	0

Then N is an nLA-ring. We define a fuzzy subset $f:N\to[0,1]$ by

$$f(x)=\begin{cases}1 & ;x\in\{0\}\\ 0.9 & ;x\in\{4\}\\ 0 & ;\text{otherwise.}\end{cases}$$

It is easy to see that f is a fuzzy left ideal of N. But f is not a fuzzy right ideal of N.

3. Fuzzy Sets

The results of the following theorems seem to play an important role to study fuzzy subsets in nLA-rings; these facts will be used frequently and normally we shall make no reference to this Lemmas.

Lemma 3.1 Let N be an nLA-ring. If f,g,h are fuzzy subsets of N, then $(f\circ g)\circ h=(h\circ g)\circ f$.

Proof. Assume that f,g,h are fuzzy subsets of N. Let $x\in N$. Then

$$\begin{aligned}(f\circ g)\circ h(x) &= \bigcup_{x=yz} min\{(f\circ g)(y),h(z)\}\\ &= \bigcup_{x=yz} min\left\{\bigcup_{y=ab} min\{f(a),g(b)\},h(z)\right\}\\ &= \bigcup_{x=(ab)z} min\{min\{f(a),g(b)\},h(z)\}\\ &= \bigcup_{x=(zb)a} min\{min\{h(z),g(b)\},f(a)\}\\ &\le \bigcup_{x=(zb)a} min\left\{\bigcup_{zb=cd} min\{h(c),g(d)\},f(a)\right\}\\ &= \bigcup_{x=(zb)a} min\{(h\circ g)(zb),f(a)\}\\ &= \bigcup_{x=wa} min\{(h\circ g)(w),f(a)\}\\ &= (h\circ g)\circ f(x).\end{aligned}$$

Thus $(f\circ g)\circ h\subseteq(h\circ g)\circ f$. Similarly $(h\circ g)\circ f\subseteq(f\circ g)\circ h$ and hence $(f\circ g)\circ h=(h\circ g)\circ f$.

If N is an nLA-ring and $F(N)$ is the collection of all fuzzy subsets of N, then $(F(N),\circ)$ is an LA-semigroup.

Lemma 3.2 Let N be an nLA-ring with left identity. If f,g,h are fuzzy subsets of N, then $f\circ(g\circ h)=g\circ(f\circ h)$.

Proof. It is straightforward by Theorem 3.1.

Lemma 3.3 If N is an nLA-ring with left identity and If f,g,h,k are fuzzy subsets of N, then $(f\circ g)\circ(h\circ k)=(k\circ h)\circ(g\circ f)$.

Proof. Assume that f,g,h,k are fuzzy subsets of N. Let $x\in N$. Then

$$\begin{aligned}&(f\circ g)\circ(h\circ k)(x)\\ &= \bigcup_{x=yz}\min\{(f\circ g)(y),(h\circ k)(z)\}\\ &= \bigcup_{x=yz}\min\left\{\bigcup_{y=ab}\min\{f(a),g(b)\},\bigcup_{z=cd}\min\{h(c),k(d)\}\right\}\\ &= \bigcup_{x=(ab)(cd)}\min\{\min\{f(a),g(b)\},\min\{h(c),k(d)\}\}\\ &= \bigcup_{x=(dc)(ba)}\min\{\min\{k(d),h(c)\},\min\{g(b),f(a)\}\}\\ &\le \bigcup_{x=(dc)(ba)}\min\left\{\bigcup_{ba=kt}\min\{k(k),h(t)\},\bigcup_{dc=mn}\min\{h(m),k(n)\}\right\}\\ &= \bigcup_{x=(dc)(ba)}\min\{k\circ h(ba),g\circ f(dc)\}\\ &= \bigcup_{x=uv}\min\{k\circ h(u),g\circ f(v)\}\\ &= (k\circ h)\circ(g\circ f)(x).\end{aligned}$$

Thus $(f\circ g)\circ(h\circ k)\subseteq(k\circ h)\circ(g\circ f)$. Similarly, $(k\circ h)\circ(g\circ f)\subseteq(f\circ g)\circ(h\circ k)$ and hence $(f\circ g)\circ(h\circ k)=(k\circ h)\circ(g\circ f)$.

Theorem 3.4 Let f be a fuzzy subset of an nLA-ring N. Then f is a fuzzy nLA-subring of N if and only if $f(x-y)\ge min\{f(x),f(y)\}$ and $f\circ f\subseteq f$ for all $x,y\in N$.

Proof. $(\Rightarrow)$ Suppose that f is a fuzzy nLA-subring of N. Let $x\in N$. If $f\circ f(x)=0$, then $f\circ f\subseteq f$. Otherwise

$$\begin{aligned}f\circ f(x) &= \bigcup_{x=yz} min\{f(y),f(z)\}\\ &\le \bigcup_{x=yz} f(yz)\\ &= f(x).\end{aligned}$$

Thus $f\circ f\subseteq f$.

$(\Leftarrow)$ Assume that $f\circ f\subseteq f$ and $f(x-y)\ge min\{f(x),f(y)\}$, for all $x,y\in N$. Let $x,y\in N$. Then by Definition of union, we have

$$\begin{aligned}f(xy) &\ge f\circ f(xy)\\ &= \bigcup_{xy=ab} min\{f(a),f(b)\}\\ &\ge min\{f(x),f(y)\}.\end{aligned}$$

This implies that f is a fuzzy nLA-subring of N.

Theorem 3.5 Let f be a fuzzy nLA-subring of an nLA-ring N. Then f is a fuzzy left ideal of N if and only if $N\circ f\subseteq f$.

Proof. $(\Rightarrow)$ Suppose that f is a fuzzy left ideal of N. Let $x\in N$. If $N\circ f(x)=0\le f(x)$, then $N\circ f\subseteq f$. Then by Definition of fuzzy left ideal, we get

$$N\circ f(x) = \bigcup_{x=yz} min\{N(y),f(z)\}$$

$$\begin{aligned} &= \bigcup_{x=yz} min\{1, f(z)\} \\ &= \bigcup_{x=yz} f(z) \\ &\le \bigcup_{x=yz} f(yz) \\ &= f(x). \end{aligned}$$

Thus $N \circ f \subseteq f.$

($\Leftarrow$) Suppose that $N \circ f \subseteq f$. Let $x \in N$. Then by Definition of union, we have

$$\begin{aligned} f(xy) &\ge N \circ f(xy) \\ &= \bigcup_{xy=ab} min\{N(a), f(b)\} \\ &\ge min\{N(a), f(y)\} \\ &= min\{1, f(y)\} \\ &= f(y). \end{aligned}$$

This implies that f is a fuzzy left ideal of N.

Theorem 3.6 Let N be an nLA-ring N. Then $N \circ N = N$.

Proof. Let $x \in N$. Then

$$\begin{aligned} N \circ N(x) &= \bigcup_{x=yz} min\{N(y), N(z)\} \\ &= \bigcup_{x=yz} min\{1,1\} \\ &= 1 \\ &= N(x). \end{aligned}$$

This implies that $N \circ N = N$.

Theorem 3.7 Let I be a nonempty subset of an nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is an nLA-subring of N if and only if f_I is a fuzzy nLA-subring of N.

Proof. ($\Rightarrow$) Suppose that I is an nLA-subring of N. Let $x, y \in N$. If $x \notin I$ or $y \notin I$, then $f_I(x) = 0$ or $f_I(y) = 0$ so that

$$f_I(xy) \ge 0 = min\{f_I(x), f_I(y)\}$$

and

$$f_I(x-y) \ge 0 = min\{f_I(x), f_I(y)\}.$$

If $x, y \in I$, then $f_I(x) = 1$ and $f_I(y) = 1$ so that $f_I(xy) = 1 = min\{f_I(x), f_I(y)\}$ and

$$f_I(x-y) = 1 = min\{f_I(x), f_I(y)\}.$$

Therefore f_I is a fuzzy nLA-subring of N.

($\Leftarrow$) Assume that f_I is a fuzzy nLA-subring of N. Let $x, y \in I$. Since

$$f_I(xy) \ge min\{f_I(x), f_I(y)\} = \{1,1\} = 1$$

and

$$f_I(x-y) \ge min\{f_I(x), f_I(y)\} = \{1,1\} = 1$$

this implies $f_I(xy) = 1$ and $f_I(x-y) = 1$. Thus $xy, x-y \in I$. Hence I is an nLA-subring of N.

Theorem 3.8 Let I be a nonempty subset of an nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is a left ideal of N if and only if f_I is a fuzzy left ideal of N.

Proof. ($\Rightarrow$) Suppose I is a left ideal of N. By Theorem 3.7, we get f_I is a fuzzy nLA-subring of N. Let $x, y \in N$. If $y \notin I$, then $f_I(y) = 0$ so that $f_I(xy) \ge 0 = f_I(y)$. If $y \in I$, then $xy \in I$ since I is a left ideal of N. This implies that $f_I(xy) = 1 = f_I(y)$. Thus f_I is a fuzzy left ideal of N.

($\Leftarrow$) Assume that f_I is a fuzzy left ideal of N. By Theorem 3.7, we get I is an nLA-subring of N. Let $r \in N$ and $x \in I$. Since $f_I(rx) \ge f_I(x) = 1$, we get $f_I(rx) = 1$. This implies that $rx \in I$ and hence I is a left ideal of N.

Theorem 3.9 Let I be a nonempty subset of an nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is a right ideal of N if and only if f_I is a fuzzy right ideal of N.

Proof. ($\Rightarrow$) Suppose I is a right ideal of N. By Theorem 3.7, we get f_I is a fuzzy nLA-subring of N. Let $x, y, z \in N$. If $x \notin I$, then $f_I(x) = 0$ so that $f_I((x+y)z-yz)\geq 0=f_I(x)$. If $x\in I$, then $(x+y)z-yz\in I$ since I is a right ideal of N. This implies that

$$f_I((x+y)z-yz)=1=f_I(x).$$

Thus f_I is a fuzzy right ideal of N.

$(\Leftarrow)$ Assume that f_I is a fuzzy right ideal of N. By Theorem 3.7, we get I is an nLA-subring of N. Let $y,z\in N$ and $x\in I$. Since $f_I((x+y)z-yz)\geq f_I(x)=1$, we get $f_I((x+y)z-yz)=1$. This implies that $(x+y)z-yz\in I$ and hence I is a right ideal of N.

Theorem 3.10 Let I be a nonempty subset of an nLA-ring N and $f_I:N\to[0,1]$ be a fuzzy subset of N such that

$$f_I(x)=\begin{cases}1 & ;x\in I\\ 0 & ;\text{otherwise.}\end{cases}$$

Then I is an ideal of N if and only if f_I is a fuzzy ideal of N.

Proof. It is straightforward by Theorem 3.8 and Theorem 3.9.

Theorem 3.11 Let I be a nonempty subset of an nLA-ring $N, m\in(0,1]$ and f_I be a fuzzy set of N such that

$$f_I(x)=\begin{cases}m & ;x\in I\\ 0 & ;\text{otherwise.}\end{cases}$$

Then the following properties hold.

(1) I is an nLA-subring of N if and only if f_I is a fuzzy nLA-subring of N.

(2) I is a left ideal of N if and only if f_I is a fuzzy left ideal of N.

(3) I is a right ideal of N if and only if f_I is a fuzzy right ideal of N.

(4) I is an ideal of N if and only if f_I is a fuzzy ideal of N.

Proof. It is straightforward by Theorem 3.10.

Definition 3.12 Let f be a fuzzy subset of nLA-ring N and $t\in(0,1]$. Then the set

$$U(f,t):=\{x\in N: f(x)\geq t\}$$

is called the level set of f.

Lemma 3.13 Let f be a fuzzy subset of an nLA-ring N. Then f is a fuzzy nLA-subring of N if and only if $U(f,t)(\neq\varnothing)$ is an nLA-subring of N, for all $t\in(0,1]$.

Proof. It is straightforward by Theorem 3.11.

Theorem 3.14 Let f be a fuzzy subset of an nLA-ring N. Then f is a fuzzy left ideal (right ideal, ideal) of N if and only if $U(f,t)(\neq\varnothing)$ is a left ideal (right ideal, ideal) of N, for all $t\in(0,1]$.

Proof. It is straightforward by Theorem 3.11.

Lemma 3.15 Let f be a fuzzy nLA-subring of an nLA-ring N. Then

1. $f(0)\geq f(x)$,
2. $f(-x)=f(x)$, for all $x\in N$.

Proof. 1. Let $x\in N$. Since f is a fuzzy nLA-subring of N, we get

$$\begin{aligned} f(0) &= f(x-x)\\ &\geq min\{f(x),f(x)\}\\ &= f(x). \end{aligned}$$

This implies that $f(0)\geq f(x)$, for all $x\in N$.

2. Let $x\in N$. Then

$$f(-x)=f(0-x)\geq\min\{f(0),f(x)\}=f(x)$$

so $f(-x)\geq f(x)$. Consider

$$\begin{aligned} f(x) &= f(0-(-x))\\ &\geq \min\{f(0),f(-x)\}\\ &\geq \min\{f(0),f(x)\}\\ &= f(x). \end{aligned}$$

Therefore $f(-x)=f(x)$, for all $x\in N$.

Proposition 3.16 Let f be a fuzzy nLA-subring of an nLA-ring N. If $f(x-y)=f(0)$, then $f(x)=f(y)$, for all $x,y\in N$.

Proof. Let $x,y\in N$ and $f(x-y)=f(0)$. Since f is a fuzzy nLA-subring of N, we get

$$\begin{aligned} f(x) &= f(0+x)\\ &= f((y-y)+x)\\ &= f((x-y)+y)\\ &= f((x-y)-(-y)) \end{aligned}$$

$$
\begin{aligned}
&\geq \min\{f(x-y), f(-y)\} \\
&= \min\{f(0), f(y)\} \\
&= f(y).
\end{aligned}
$$

By Lemma 3.15, we have

$$
\begin{aligned}
f(y) &= f(0+y) \\
&= f((x-x)+y) \\
&= f((y-x)+x) \\
&= f((y-x)+(0+x)) \\
&= f((x+0)+(-x+y)) \\
&= f((x+0)-(-(-x+y))) \\
&\geq \min\{f(x+0), f(-(-x+y))\} \\
&\geq \min\{\min\{f(x), f(0)\}, f(x-y)\} \\
&= \min\{f(x), f(0)\} \\
&= f(x).
\end{aligned}
$$

Hence $f(x)=f(y)$, for all $x,y\in N$.

Theorem 3.17 Let f be a fuzzy subset of an nLA-ring N. If f is a fuzzy nLA-subring of N, then $N_f=\{x\in N : f(x)=0\}$ is an nLA-subring of N.

Proof. Assume that f is a fuzzy nLA-subring of N. Let $x,y\in N_f$. Then $f(x)=f(0)$ and $f(y)=f(0)$. By Lemma 3.15, we have

$$
\begin{aligned}
f(x-y) &\geq min\{f(x), f(y)\} \\
&= min\{f(0), f(0)\} \\
&= f(0)
\end{aligned}
$$

and

$$
\begin{aligned}
f(xy) &\geq min\{f(x), f(y)\} \\
&= min\{f(0), f(0)\} \\
&= f(0).
\end{aligned}
$$

By Lemma 3.15, we get $f(x-y)=f(0)$ and $f(xy)=f(0)$ which implies that $x-y, xy\in N_f$. Hence N_f is an nLA-subring of N.

Theorem 3.18 Let f be a fuzzy subset of an Γ-nLA-ring N. If f is a fuzzy left ideal of N, then $N_f=\{x\in N : f(x)=0\}$ is a left ideal of N.

Proof. Assume that f is a fuzzy left ideal of N. By Theorem 3.17, we get N_f is an nLA-subring of N. Let $r\in N$ and $x\in N_f$. Then $f(x)=f(0)$ so that $f(rx)\geq f(x)=f(0)$. By Lemma 3.15, we get $f(rx)=f(0)$ which implies that $rx\in N_f$. Hence N_f is a left ideal of N.

Theorem 3.19 Let f be a fuzzy subset of an Γ-nLA-ring N. If f is a fuzzy right ideal of N, then $N_f=\{x\in N : f(x)=0\}$ is a right ideal of N.

Proof. Assume that f is a fuzzy right ideal of N. By Theorem 3.17, we get N_f is an nLA-subring of N. Let $y,z\in N$ and $x\in N_f$. Then $f(x)=f(0)$ so that

$$f((x+y)z-yz)\geq f(x)=f(0).$$

By Lemma 3.15, we get $f((x+y)z-yz)=f(0)$ which implies that $(x+y)z-yz\in N_f$. Hence N_f is a right ideal of N.

Theorem 3.20 Let f be a fuzzy subset of an Γ-nLA-ring N. If f is a fuzzy ideal of N, then $N_f=\{x\in N : f(x)=0\}$ is an ideal of N.

Proof. It is straightforward by Theorem 3.18 and Theorem 3.19.

4. Conclusions

Many new classes of nLA-rings have been discovered recently. All these have attracted researchers of the field to investigate these newly discovered classes in detail. This article investigates the fuzzy set, fuzzy left and fuzzy right ideals in nLA-rings. We show that a nonempty subset I of an nLA-ring N is an nLA-subring (left ideal, right ideal, ideal) of N if and only if f_I (tf_I) is a fuzzy nLA-subring (fuzzy left ideal, fuzzy right ideal, fuzzy ideal) of N. Finally we show that if f is a fuzzy nLA-subring (fuzzy left ideal, fuzzy right ideal, fuzzy ideal) of N, then $N_f=\{x\in N : f(x)=0\}$ is an nLA-subring (left ideal, right ideal, ideal) of N.

References

[1] Zadeh LA. Fuzzy sets. **Information and control**. 1965; **8**: 338-353.
[2] Abou Zaid S. On fuzzy subnear-rings and ideals. **Fuzzy Sets and Systems**. 1991; **44**: 139-146.

[3] Narayanan AL, Manikantan T. $(\in, \in \vee_q)$-fuzzy subnear-rings and $(\in, \in \vee_q)$-fuzzy ideals of near-rings. **J. Appl. Math. Computing.** 2005; **18**: 419-430.
[4] Kim SD, Kim HS. On fuzzy ideals of near-rings. **Bulletin of the Korean Mathematical Society**. 1996; **33**: 593-601.
[5] Davvaz B. Fuzzy ideals of near-rings with interval valued membership functions. **Journals of Sciences, Islamic republic of Iran**. 2001; **12** (2): 171-175.
[6] Kamran S. **Structural properties of LA-semigroups.** [dissertation]. Quaid-i-Azam University, Islamabad; 1987.
[7] Yusuf SM. On left almost ring. **Proceeding of 7th International Pure Mathematics Conference**; 2006.
[8] Shah T, Rehman F, Raees M. On near left almost rings. **International Mathematical Forum**. 2011; **6** (23): 1103-1111.