Journal of
Thai Interdisciplinary Research

Volume 13, Number 2, Pages 18–24

Design of decentralized PID controller with the root locus method based on inverted decoupling for a TITO system

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Abstract

Decentralized control is widely applied in several industrial plants and one of the decentralized techniques is the decoupled method used to reduce the interaction of diagonal loops and can be minimized TITO system into SISO system. This paper presents the decentralized PID controller design for TITO system using root locus method. An inverted decoupling approach is analyzed by an application of Wood–Berry distillation column process, when a FOPDT model of the process is considered. The results showed that by using the root locus method, the proper control parameters can be determined and the designed control system can regulate the process variables of TITO process. The designed decentralized PID controller based on inverted decoupling gave an effectiveness to maintain the controlled variables.

Keywords: decentralized PID controller, inverted decoupling, root locus, TITO

1. Introduction

Many industrial plants are completely integrated processes that present non-linear behaviour and complex dynamic properties. Various industrial controlled problems have multiple manipulated and controlled variables that the problem of tuning is more complicated due to interactions of multi loops making the design of efficient controller challenging [1, 2]. The interactive multi-input multi-output (MIMO) control systems can be designed by centralized or decentralized control method [3, 4] but, however, in most of the industrial processes, decentralized control is widely used instead of multivariable centralized control because the decentralized controller is simple to design and easy to tune, implement and maintain [5, 6]. Decentralized control design with decoupling technique is most regularly preferred for MIMO processes. Decoupling, which is one of the advanced regulatory control strategies, is designed to transform the multivariable process to a diagonal dominant plant allowing individual design of controller for each loop [7]. The study on decoupling techniques has attracted the attention of many researchers over the years. In case of control design of two-input two-output (TITO) process, many researchers investigated the advantages and limitations of each decoupling techniques mainly including ideal decoupling [8, 9], simplified decoupling [9, 10], normalized decoupling [11, 12] and inverted decoupling [7, 9, 10, 13–15]. Ideal decoupling based on inverse transfer function matrix of the plant, which is not commonly implemented, is design to attain perfect decoupling and a simple apparent process [7, 10]. Simplified decoupling is wildly used in practice and easy to implement but the difficulty in controller tuning is its limitation because of the complex of the sum of transfer functions of its elements [9]. Normalized decoupling is easy to understand and realizes simplicity by reasonable approximation [11, 12]. Among the many types of decoupling techniques, inverted decoupling is one of the most significant types of dynamic decoupling techniques [16] although it is rarely implemented; it presents the main advantage of both the simplified and ideal decoupling methods [9]. Gagnon et al. [9] also compared the advantages and disadvantages of simplified, ideal and inverted decoupling techniques. As there are many advantages of inverted decoupling method, this research focuses on the inverted decoupling technique for TITO system of the well-known Wood–Berry distillation column process [17] which is one of the most prominent case studies to investigate various methods of controller design.

In case of controller design for the Wood-Berry process, the decentralized PI controller design method [6, 18] and the decentralized PID controller design method [3, 6, 18–20] for TITO system with decoupler have been proposed. Additionally, the inverted decoupling based control has been applied for many researches [7, 16, 21–28]. Hajare and Patre [3] proposed the decentralized PID controller for TITO systems using characteristic ratio assignment (CRA) that the interactions of loops are reduced using ideal decoupler reduced to first order plus dead time (FOPDT) model. Maghade and Patre [6] investigated a decentralized PI/PID controller design method based on gain and phase margin specifications for TITO interactive

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DOI 10.14456/jtir.2018.14

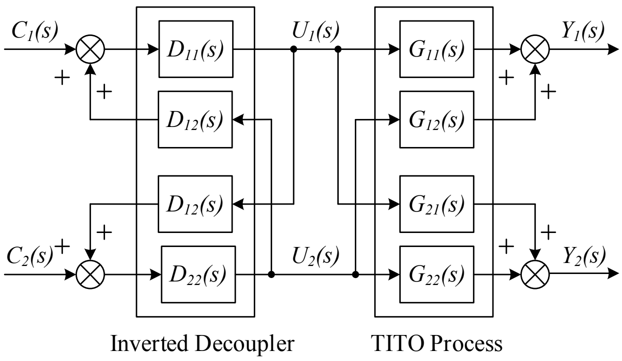

Figure 1 Inverted decoupling of a TITO process

processes. Wang et al. [18] developed an auo-tuning of simple lead-lag decoupler plus decentra-lized PI/PID controller for effective control of TITO processes. Vázquez and Morilla [19] compared the time response of tuning decentralized PID controllers with and without decouplers. Jin et al. [20] proposed a design of decentralized PID controller using internal model control based on decoupler for TITO system with time delay. There are many literatures proposed to use inverted decoupling method for controller design of TITO process by using various design methodologies such as active disturbance rejection control for multi-variable systems [7], internal model control [16, 21], Smith predictor [22], and Multiple integration (MI) and Multiple optimum (MO) techniques [23]. Mekki [24] studied a decentralized PI controller combined with an inverted decoupling. Garrido et al. [25, 26] proposed centralized PID control by inverted decoupling. Li and Chen [27] proposed the fractional order TITO process with ideal, simplified and inverted decoupling.

As mentioned above, a design of decentralized PID controller with root locus method based on inverted decoupling for TITO system is performed in this research. The main objectives of this research are to investigate the decentralized PID controller design using root locus method based on inverted decoupling control by the application for the well-known Wood-Berry distillation column process and to compare the results of controller design and time response with other researches. In section 2, the TITO process combined with inverted decoupling is presented. Root locus method for controller design is proposed in section 3. Simulation results of Wood-Berry process case study are given by section 4, followed by conclusion in section 5.

2. Inverted Decoupling

A block diagram of inverted decoupling structure of TITO process is described in figure 1, when $G(s) = [G_{ij},(i,j=1,2)]_{2\times 2}$ is the TITO process and $D(s) = [D_{ij},(i,j=1,2)]_{2\times 2}$ is the decoupler [21]. Inverted decouplers were added into TITO process in order to eliminate the loop interactions to be SISO process that product of transfer function of $G(s)D(s)$ is a diagonal transfer matrix $T(s)$ as presented in Equation (1).

The product of plant and decoupler matrix to be a diagonal transfer function matrix of decoupled process can be given by

$$\begin{bmatrix} T_{11}(s) & 0 \\ 0 & T_{22}(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \quad (1)$$

The inverted decouplers are to accomplish the ideally decoupled performance as presented in Equation (1), using simplified decoupling elements in Equation (2) [27].

$$\begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{G_{12}(s)}{G_{11}(s)} \\ -\dfrac{G_{21}(s)}{G_{22}(s)} & 1 \end{bmatrix} \quad (2)$$

Then, Equation (1) can be written as

$$T(s) = \frac{1}{1-\dfrac{G_{12}(s)G_{12}(s)}{G_{11}(s)G_{22}(s)}} \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & -\dfrac{G_{12}(s)}{G_{11}(s)} \\ -\dfrac{G_{21}(s)}{G_{22}(s)} & 1 \end{bmatrix}$$
$$= \frac{G_{11}(s)G_{22}(s)}{G_{11}(s)G_{22}(s)-G_{12}(s)G_{21}(s)} \begin{bmatrix} G_{11}(s)-\dfrac{G_{12}(s)G_{21}(s)}{G_{22}(s)} & 0 \\ 0 & G_{22}(s)-\dfrac{G_{12}(s)G_{21}(s)}{G_{11}(s)} \end{bmatrix} \quad (3)$$

The term of dead time in the transfer function of $T(s)$ in Equation (3) is approximated by first-order Taylor series approximation as given by

$$e^{-\tau_d s} = \frac{1}{\tau_d s + 1} \quad (4)$$

Consequently, the controlled variables of the decoupled system can be given by

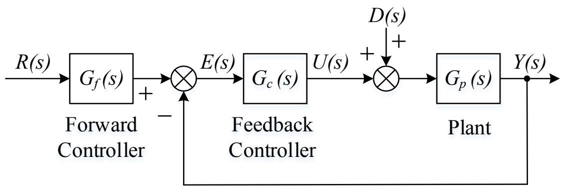

Figure 2 2-DOF control system

$$Y_1(s) = \frac{G_{11}(s)G_{22}(s)}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)}\left(G_{11}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{22}(s)}\right) \tag{5}$$

$$Y_2(s) = \frac{G_{11}(s)G_{22}(s)}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)}\left(G_{22}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{11}(s)}\right) \tag{6}$$

The most significant advantages of the proposed inverted decoupling, amongst the others are that decouplers are simple to implementation and apparent processes for SISO controllers [23].

3. Root Locus Technique

In this research, root locus technique is applied for designing the control parameters of the inverted decoupling TITO system. A block diagram of two degree of freedom (2-DOF) control system structure consisting of the feedback controller $G_c(s)$ and forward controller $G_f(s)$ is demonstrated in figure 2.

Using Root locus technique for controller design, the characteristic of transient response and steady state response can be explained as follows [28].

1) The characteristic of transient response can be described in term of maximum percent overshoot (M_p).

2) The characteristic of steady state response can be described in term of settling time (t_s).

The method for designing the satisfying response at the transient state and steady state can be applied by the following steps.

Step 1: Determine the damping ratio (ξ), natural frequency (ω_n) by considering the characteristic of transient response and steady state response and dominant poles (s_d) from Equation (7).

$$M_p = 100e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}},\ t_s^{(\pm 2\%)} = \frac{4}{\xi\omega_n},\ s_d = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \tag{7}$$

Step 2: Find the summation of angle at s_d of the open loop system $G_p(s)G_c(s)$ by graphical method or arithmetical method and then consider the essential angle of $\angle(s_d + z_c)$ in order to obtain the summation of angle according to the system condition as shown in Equation (8).

$$\sum(\theta_z + \theta_{zc}) + \sum\theta_p = -(2k+1)\pi,\ \ k = 0,1,\ldots,n \tag{8}$$

Step 3: Calculate the gain K_c of the controller by using the root locus technique as given by Equation (9).

$$K_c = K_{sd} = \frac{1}{\left|G_p(s_d)G_c(s_d)\right|} \tag{9}$$

Step 4: To obtain the satisfying response by setting a unit step input signal, therefore, the forward controller is added as shown in figure 2 and transfer function of this controller can be written in Equation (10).

$$G_f(s) = \frac{z_c}{(s+z_c)} \tag{10}$$

4. Simulation Results and Discussion

The control system for decentralized PID controller with inverted decoupling of TITO process was designed as illustrated in figure 3. This research considered the Wood and Berry distillation column process [17] that was introduced the transfer function model of a pilot-scale distillation column composed of an eight-tray plus re-boiler separating methanol and water which is typical TITO process with strong interaction and significant time delays. Consequently, the Wood–Berry binary distillation column process is considered as the TITO system that has been extensively studied by most of the researchers. This process can be written the transfer function matrix as given by Equation (11).

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{11}$$

From Equation (2), the inverted decoupler can be determined as Equation (12).

$$D(s) = \begin{bmatrix} 1 & \dfrac{1.477(16.7s+1)e^{-2s}}{(21s+1)} \\ \dfrac{0.34(14.4s+1)e^{-4s}}{(10.9s+1)} & 1 \end{bmatrix} \tag{12}$$

The TITO process was reduced to be SISO system by using inverted decoupler. The each loop transfer

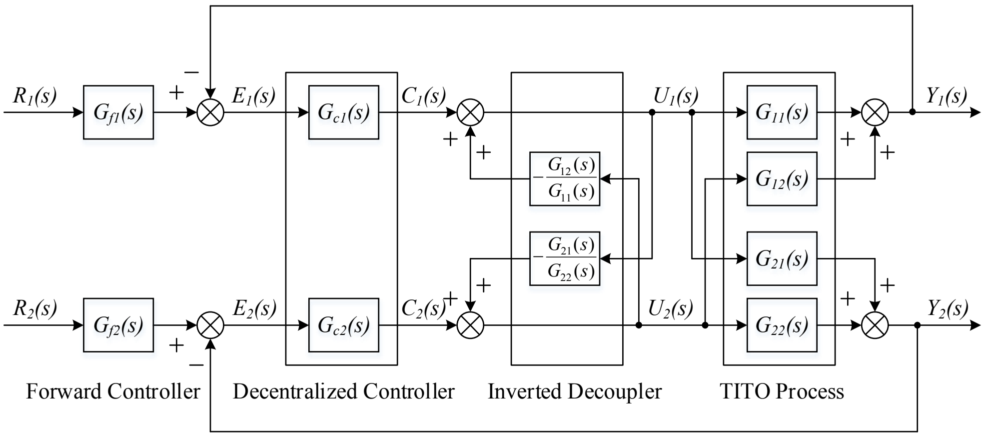

Figure 3 Decentralized PID controller based on inverted decoupling for TITO system

function of SISO system consisting of loop-1 (y1-r1) and loop-2 (y2-r2) can be written as Equation (13) and Equation (14), respectively.

$$T_{11}(s)=\frac{1}{1-\frac{0.5017(16.7s+1)(14.4s+1)}{(21s+1)(10.9s+1)}}\left(\frac{12.8}{(16.7s+1)}e^{-s}-\frac{6.43(14.4s+1)}{(10.9s+1)(21s+1)}e^{-7s}\right) \tag{13}$$

$$T_{22}(s)=\frac{1}{1-\frac{0.5017(16.7s+1)(14.4s+1)}{(21s+1)(10.9s+1)}}\left(-\frac{19.4}{(14.4s+1)}e^{-3s}-\frac{9.7453(16.7s+1)}{(10.9s+1)(21s+1)}e^{-9s}\right) \tag{14}$$

Therefore, Equation (13) and Equation (14) can be estimated as the FOPDT model obtained by frequency response [29] as given by Equation (15) and Equation (16), respectively.

$$T_{11}(s)=\frac{12.7835e^{-1.002s}}{(16.671s+1)}\cong\frac{12.7835}{(16.671s+1)(1.002s+1)} \tag{15}$$

$$T_{22}(s)=\frac{-19.3753e^{-1.0186s}}{(14.3753s+1)}\cong\frac{-19.3753}{(14.3753s+1)(1.0186s+1)} \tag{16}$$

Root locus technique was used to determine controller parameters of TITO process reduced to be SISO system and the transfer functions in Equation (15) and Equation (16) for loop-1 and loop-2, respectively, were used to design PID controller. The 2-DOF control strategy was applied in this paper. In the control design, a graphical root locus technique was used to determine the PID controller parameters composed of K_d (derivative gain), K_p (proportional gain) and K_i (integral gain). For both loops of reduced SISO system, the settling time of 100 sec and the percent overshoot of 5% were set as the conditional requirement when damping factor (ξ) is 0.6901, natural frequency (ω_n) is 0.058 and dominant poles (s_d) is $-0.04\pm j0.0419$. Accordingly, the transfer function matrix of feedback controllers and forward controllers of loop-1 and loop-2 for the proposed method can be written as Equation (17).

$$G_c(s)=\begin{bmatrix}\frac{(0.4609s^2+0.5387s+0.0774)}{s} & 0\\ 0 & \frac{(-0.1213s^2-0.1583s-0.0384)}{s}\end{bmatrix}$$

$$G_f(s)=\begin{bmatrix}\frac{0.1678}{(s+0.1678)} & 0\\ 0 & \frac{0.3218}{(s+0.3218)}\end{bmatrix} \tag{17}$$

Therefore, the PID controller parameters of the Wood-Berry distillation column combined inverted decoupler process by designing of the decentralized PID controller using root locus method can be calculated. For loop-1, the PID controller parameters are K_d = 0.4609, K_p = 0.5387, and K_i = 0.0774. For loop-2, the PID controller parameters are K_d = – 0.1213, K_p = – 0.1583, and K_i = – 0.0384.

For this case study, the control performance of the proposed controller is compared with the controller design presented by Maghade and Patre [6] (G_cMP) and Tavakoli et al. [30] ($G_cTavakoli$) using the decoupling as referred in Equation (12) that the parameters of controller are given by Equation (18) and Equation (19), respectively. Controller (G_cMP) is introduced in literature with desired Gain Margin (GM) of 3, Phase Margin (PM) of 60° and Controller ($G_cTavakoli$) considering GM ≥ 3 and PM ≥ 60° as the robustness constraints, the optimal PI tuning parameters are calculated.

$$G_cMP=\begin{bmatrix}\frac{0.9733+\frac{0.0881}{s}+2.6887s}{(5.5252s+1)} & 0\\ 0 & \frac{-0.3134-\frac{0.0304}{s}-0.8070s}{(5.1499s+1)}\end{bmatrix} \tag{18}$$

$$G_cTavakoli=\begin{bmatrix}0.41+\frac{0.074}{s} & 0\\ 0 & -0.120-\frac{0.024}{s}\end{bmatrix} \tag{19}$$

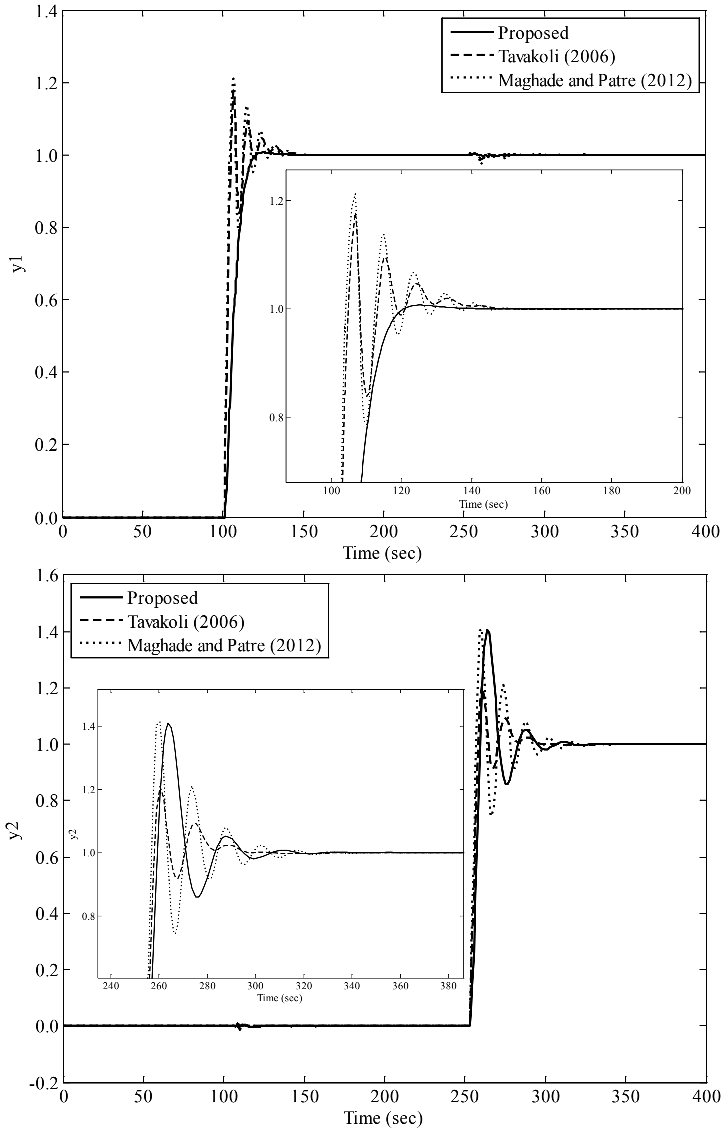

Figure 4 Unit step response a) the loop-1 (y1-r1) and b) the loop-2 (y2-r2)

Table 1 Settling time, percent overshoot and maximum control signal of loop-1 and loop-2

Method	Loop-1 (y1-r1)			Loop-2 (y2-r2)		
	$t_s(sec)$	$M_p(\%)$	$t_s(sec)$	$M_p(\%)$	$t_s(sec)$	$M_p(\%)$
Proposed Method	20	1.0	0.230	43.0	40.82	-0.1585
Tavakoli (2006)	27	17.8	0.474	41.0	20.80	-0.1400
Maghade and Patre (2012)	34	21.0	0.562	53.5	41.20	-0.1895

For simulation, a unit change to the setpoint inputs for loop-1 at t =100 sec and for loop-2 at t = 250 sec is added. The output responses and control signal of both loops are shown in figures 4 and 5, respectively. For loop-1 response, it is illustrated that the proposed controller design method achieves a good performance in term of faster settling time and smaller percent overshoot and maximum control signal (u_{max}). The result of output response for proposed controller design can attain the requirement of settling time and percent overshoot. For loop-2 response, the proposed controller design gave better control performance than control design introduced by Maghade and Patre [6] and the settling time and control signal can be competitive with the performance of controller design introduced by Tavakoli et al. [30] although the percent overshoot of this proposed method is higher than that of Tavakoli's control design as presented in Table 1.

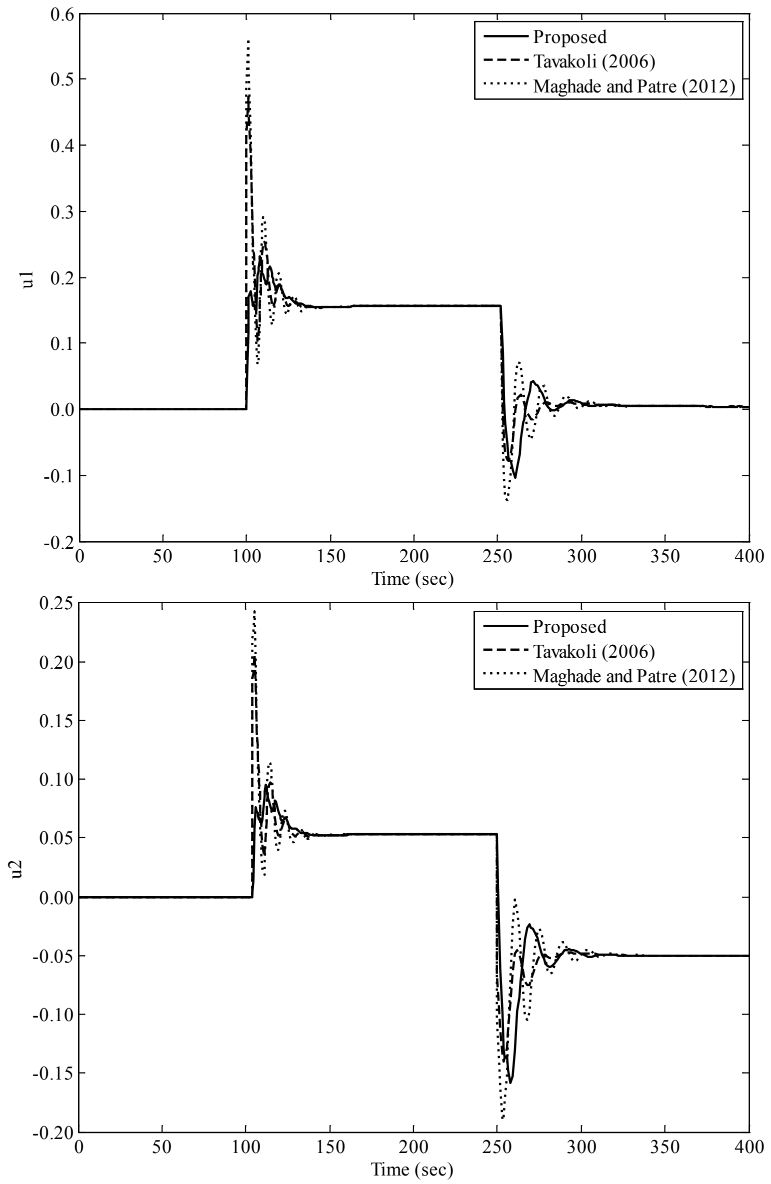

Figure 5 Control signal response a) the controller 1 (u1-r1) and b) the controller 2 (u2-r2)

5. Conclusion

This paper proposed a decentralized PID controller design for TITO process using root locus technique. The TITO process was combined with inverted decoupler due to its many advantages for eliminating loop interactions of TITO process to be SISO system. The each loop of SISO system was approximated as FOPDT model to use for designing the PID controller. As the PID controller parameters of the case study of Wood-Berry distillation column process were designed, the results showed that the proposed controller can achieve a good performance for regulating the process variables.

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